Proof Theory an Automated Theorem Proving 2013
Exercises
Week 1

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1 Natural deduction

Prove the following formulas using natural deduction:

1. $(\varphi \wedge \psi \to \sigma) \to (\varphi \to (\psi \to \sigma))$
2. $(\psi \to (\varphi \to \sigma)) \to (\varphi \wedge \psi \to \sigma)$
3. $(\varphi \to \psi) \to (\varphi \wedge \chi \to \psi)$
4. $\varphi \vee \psi \to \varphi \vee (\varphi \vee \psi)$
5. $\varphi \vee (\varphi \vee \psi) \to \varphi \vee \psi$
6. $\varphi \to (\psi \to \varphi \wedge \psi)$
7. $\varphi \wedge \psi \to \psi \wedge \varphi$
8. $(\varphi \wedge \psi) \vee \varphi \to \varphi$
9. $\varphi \to (\varphi \wedge \psi) \vee \varphi$
10. $(\varphi \to (\psi \wedge \chi)) \to (\varphi \to \psi) \wedge (\varphi \to \chi)$
11. $(\varphi \to \psi) \wedge (\varphi \to \chi) \to (\varphi \to (\psi \wedge \chi))$
12. $(\varphi \vee \psi \to \chi) \to ((\varphi \to \chi) \wedge (\psi \to \chi))$
13. $(\varphi \vee \psi) \wedge (\psi \to \varphi) \to \varphi$
14. $(\varphi \to \chi) \wedge (\psi \to \chi) \to (\varphi \vee \psi \to \chi)$
15. $\psi \to (\varphi \vee \psi) \wedge (\neg\varphi \vee \psi)$.

16. $(\varphi \vee \psi) \wedge (\neg\varphi \vee \psi) \to \psi$.
17. $(\varphi \wedge \psi) \vee (\neg\varphi \wedge \psi) \to \psi$.
18. $\psi \to (\varphi \wedge \psi) \vee (\neg\varphi \wedge \psi)$.
19. $\neg\neg\varphi \to \varphi$
20. $((\varphi \to \psi) \to \varphi) \to \varphi$

2 Constructions on proofs

Prove the following statements "glueing" hypothetical proofs together in an adequate fashion.

1. If $\vdash \varphi \wedge \psi$, then $\vdash \varphi$ and $\vdash \psi$.
2. If $\vdash \varphi \vee \psi$ and $\vdash \neg\varphi$, then $\vdash \psi$.
3. If $\vdash \varphi$ and $\vdash \psi$, then $\vdash \varphi \wedge \psi$.
4. If $\vdash \varphi \to \psi$ and $\vdash \neg\psi$, then also $\vdash \neg\varphi$.
5. If $\vdash \varphi \to \neg\varphi$, then $\vdash \neg\varphi$.
6. If $\vdash \varphi \to \neg\varphi$ and $\vdash \neg\varphi \to \varphi$ then $\vdash \bot$.

3 Intuitionistic logic

Give constructive proofs of the following tautologies:

1. $\varphi \to \neg\neg\varphi$
2. $\neg\neg\neg\varphi \to \neg\varphi$
3. $\neg\neg\varphi \wedge \neg\neg\psi \to \neg\neg(\varphi \wedge \psi)$