

Proof Theory and Automated Theorem Proving
2013
Exercises
Week 2a

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1 t is free for x in φ

In this exercise, we work in first order logic. Let us work with a signature¹ with constants c and d , a unary function symbol f and a binary function symbol g and a ternary relation R .

1. Give an inductive definition of terms for this signature.
2. Give an inductive definition of formula in this signature.
3. Give an inductive definition of the free variables in a term.
4. Give an inductive definition of free variables in a formula.
5. Give an inductive definition of the bound variables in a formula.
6. Give an inductive definition of t is free for x in φ .

Hint: you can define certain properties simultaneously which saves writing and repetition.

2 Predicate logic and natural deduction

Prove the following exercises using natural deduction.

1. $\exists x (\varphi(x) \rightarrow \psi) \rightarrow (\forall x \varphi(x) \rightarrow \psi)$ with $x \notin \text{FV}(\psi)$,
2. $(\forall x \varphi(x) \rightarrow \psi) \rightarrow \exists x (\varphi(x) \rightarrow \psi)$ with $x \notin \text{FV}(\psi)$,
3. $\vdash \forall x(\varphi(x) \rightarrow \psi(x)) \rightarrow (\forall x\varphi(x) \rightarrow \forall x\psi(x))$
4. $\vdash \forall x\varphi(x) \rightarrow \neg\forall x\neg\varphi(x)$

¹You can make your definitions for an arbitrary signature, or for the one given here, as you wish.