Proof Theory an Automated Theorem Proving 2013
Exercises
Week 2

Lecturer: Joost J. Joosten

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1 Predicate logic and natural deduction

Prove the following exercises using natural deduction.

1. $\vdash \forall x \varphi(x) \to \neg \exists x \neg \varphi(x)$
2. $\vdash \exists x \neg \varphi(x) \to \neg \forall x \varphi(x)$
3. $\vdash \neg \forall x \varphi(x) \to \exists x \neg \varphi(x)$

2 Constructive proofs

Give constructive natural deduction proofs of the following tautologies:

1. $\neg\neg\neg\psi \to \neg\psi$
2. $\exists x\ (\phi(x) \vee \psi(x)) \to \exists x\ \phi(x) \vee \exists x\ \psi(x)$
3. $\exists x\ \phi(x) \vee \exists x\ \psi(x) \to \exists x\ (\phi(x) \vee \psi(x))$
4. $\forall x\ (\phi(x) \to \psi) \to (\exists x\ \phi(x) \to \psi)$
5. $(\exists x\ \phi(x) \to \psi) \to \forall x\ (\phi(x) \to \psi)$
6. $\forall x\ (\phi(x) \to \psi) \to \forall x\ (\phi(x) \to \psi)$

3 Propositional logic in Tait calculus

Prove the following formulas in Tait calculus. Remember that one first has to translate the formulas below to formulas in the Tait language.

1. $(\varphi \wedge \psi \to \sigma) \to (\varphi \to (\psi \to \sigma))$
2. $(\psi \to (\varphi \to \sigma)) \to (\varphi \wedge \psi \to \sigma)$
3. $(\varphi \to \psi) \to (\varphi \wedge \chi \to \psi)$

4 Tait language

1. Make Exercise 4.3.4 from the book.
2. Argue (informal reasoning is allowed) to the effect that F and F^T are actually equivalent so that for each formula in the original language, there is one in the Tait language which is equivalent.

5 Predicate logic in Tait calculus

Prove the following exercises the Tait calculus.

1. $\exists x\ (\varphi(x) \to \psi) \to (\forall x\ \varphi(x) \to \psi)$
2. $(\forall x\ \varphi(x) \to \psi) \to \exists x\ (\varphi(x) \to \psi)$
3. $\vdash \forall x(\varphi(x) \to \psi(x)) \to (\forall x \varphi(x) \to \forall x \psi(x))$