

Proof Theory an Automated Theorem Proving
2013
Exercises
Week 6

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March 22, 2013

1 Monotone Operators versus Inductive Definitions

For this exercise, we will use notation from Section 6.2 of the book. Moreover, given monotone operator Ψ , let \mathcal{A}_Ψ denote the corresponding inductive definition as defined in the book.

1. Let Ψ be a monotone operator. Show that

$$\Phi_{(\mathcal{A}_\Psi)} = \Psi.$$

2. Let \mathcal{B} be an arbitrary inductive definition. Do we also have

$$\mathcal{A}_{(\Phi_{\mathcal{B}})} = \mathcal{B}?$$

Give a proof or a counter-example.

3. For \mathcal{A} , prove that $I(\mathcal{A}) = I(\Phi_{\mathcal{A}})$.
4. For Ψ a monotone operator, prove that $I(\Psi) = I(\mathcal{A}_\Psi)$.

2 Closure ordinal

1. Compute the closure ordinal for the inductive definition of the set of propositional formulas (codes of formulas for that matter).
2. Fill in the details of the proof of Lemma 6.3.6.