1 First exercise

Theorem 1. $\vdash \phi \to \psi \implies \vdash \Box\phi \to \Box\psi$.

Proof.

1 .	$\phi \to \psi$	given
2 .	$\Box(\phi \to \psi)$	N, a
3 .	$\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$	$[\phi/p, \psi/q]$ K
4 .	$\Box\phi \to \Box\psi$	MP, b, c

□

Theorem 2. $\Diamond\phi \to \Diamond\phi$.

2 Second exercise