

EXERCISES

JJJ

1. FIRST BATCH OF EXERCISES

- (1) We have seen the simplest form of the Curry-Howard isomorphism in class. Provide the details of the proof to the extent that for every formula A that is constructed only from propositional variables and implications we have

$$\vdash_m A \iff \exists t^A \in \mathbf{Term}_{\rightarrow} \quad \mathbf{FV}(t) = \emptyset$$

- (2) Since the combinators **S** and **K** can be typed, we can add these combinators with their rewriting rules to the simply typed lambda calculus. The type of **S** is always of the form $(\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma))$ for some concrete types α, β, γ . Likewise, the type of **K** is always of the form $\alpha \rightarrow (\beta \rightarrow \alpha)$ for some concrete types α, β . A sequence of combinators is *typable* whenever we can assign concrete types to the combinators so that the entire sequence of combinators can be assigned a type according to the typing rules.
- (a) Show that the sequence **SK** is typable.
 - (b) Show that the sequence **SKK** is typable.
 - (c) Is every sequence of **S** and **K** typable? If so, give a proof, if not, give a counter example.