AUTOMATED THEOREM PROVING

Herbrand's Theorem

Exercise 1. (a) Let $\phi = Q_1 x_1 \cdots Q_n x_n \psi$ be a firs-order formula in prenex form. Prove that if ψ is a tautology in the propositional sense, then ϕ is a tautology in the first-order sense.

(b) Prove that the implication from right to left in (a) is false.

Exercise 2. Suppose that ϕ is a first-order formula and ψ is a Skolem standard form of $\neg\phi$. Find a sufficient and necessary condition for ϕ such that the Herbrand universe of ψ is finite.

Exercise 3. Consider the set of clauses $\Phi = \{P(x), \neg P(f(y))\}$. We say that a B-interpretation I satisfies Φ, if H_I is a model of α_Φ. Then, decide whether there is a B-interpretation that satisfies Φ.

Exercise 4. Let $\Phi = \{P(x), \neg P(x) \vee Q(x, c), \neg Q(y, c)\}$. Then:

(a) Construct the Herbrand base of Φ.

(b) Give a closed semantic tree for Φ.

Exercise 5. Let $\Phi = \{P(x), Q(x, f(x)) \vee \neg P(x), \neg Q(g(y), z)\}$. Prove that the Skolem form α_Φ is unsatisfiable by finding an unsatisfiable finite set of ground instances of Φ.

Exercise 6. Explain where the proof of Herbrand's theorem fails if we allow the equality relation in first-order formulas. Give a counterexample.