Instrumentalism in mathematics
OOO

Useful results for speed-up
O

Three scenarios
OOO

Conclusions
OO

Challenges to Instrumentalism

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On instrumentalism and philosophy of science

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- Vraag niet hoe het kan maar profiteer ervan

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- In this talk: Speedup over PRA

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- Can be generalised in various directions,
  for example

$$\psi_{(n,f)} \quad \longleftrightarrow \quad \neg \exists\, p{\leq}f(\overline{n})\ \text{Proof}_T(p, \psi_{(n,f)})$$

for recursive f.

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- Technical improvements are possible: $\{\varphi_i\}_i$ provided in a simple fashion; speed-up for formulas of lower complexity; and the same sequence for both choices, can we take Orey sentences, etc.

A false statement providing speed-up

- Through the previous result, for example using $\neg\mathrm{Con}(T)$ for consistent T
- Re-prompts the recurrent issue related to Gödel 2: reasoning to the extent that the instrument really is false need not be recognised by the standards of the instrumentalist as meaningful
- Technical curiosity: can we make the speed-up by the false statement be much larger than the one through its negation?

A disprovable statement yielding speed-up

- Given T let α be so that
    - α is refutable in T, BUT
    - the smallest proof of $\neg\alpha$ in T is of αstronimical size
    - Then, $T + \alpha$ has non-recursive speed-up over T
    - Again, by considering $\{\alpha \vee \varphi_n\}_{n\in\omega}$ for suitable φ_n.

- Does this force some sort of notion of *ultra-finitism* upon the Instrumentalist?

Conclusions

- The basic intuitions behind Instrumentalism seem to have some bearing on mathematics too
- Not yet entirely clear how and how this interacts with mathematical results

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Thank you for your attention

THANKS!